Smale's mean value conjecture for finite Blaschke products

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Smale's mean value conjecture asserts that if P is a monic polynomial of degree $n \geq 2$ such that $P(0) = 0$ and $P'(0) = 1$ and $b_1, \ldots, b_{n-1}$ are its critical points. Then

$$\min_i \left| \frac{P(b_i)}{b_i P'(0)} \right| \leq \frac{n-1}{n}.$$

While the dual mean value conjecture proposed by Dubinin & Sugawa and the speaker independently asserts that

$$\max_i \left| \frac{P(b_i)}{b_i P'(0)} \right| \geq \frac{1}{n}.$$

In 2009, Dubinin and Sugawa showed that

$$\max_i \left| \frac{P(b_i)}{b_i P'(0)} \right| \geq \frac{1}{n4^n}.$$

Motivated by a dictionary between polynomials and finite Blaschke products, we study both Smale's mean value conjecture and its dual conjecture for finite Blaschke products in this talk. The result on the dual conjecture for finite Blaschke products also implies a better bound for the dual conjecture for polynomials, i.e.

$$\max_i \left| \frac{P(b_i)}{b_i P'(0)} \right| > \frac{1}{4^n}.$$

This is a joint work with Yongquan Zhang.